Mesoscopic few-body problem with short-range interactions

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IFRAF

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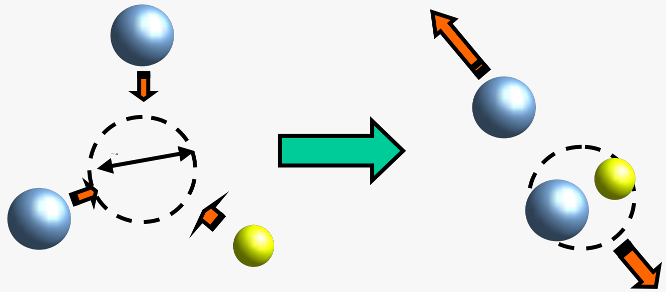
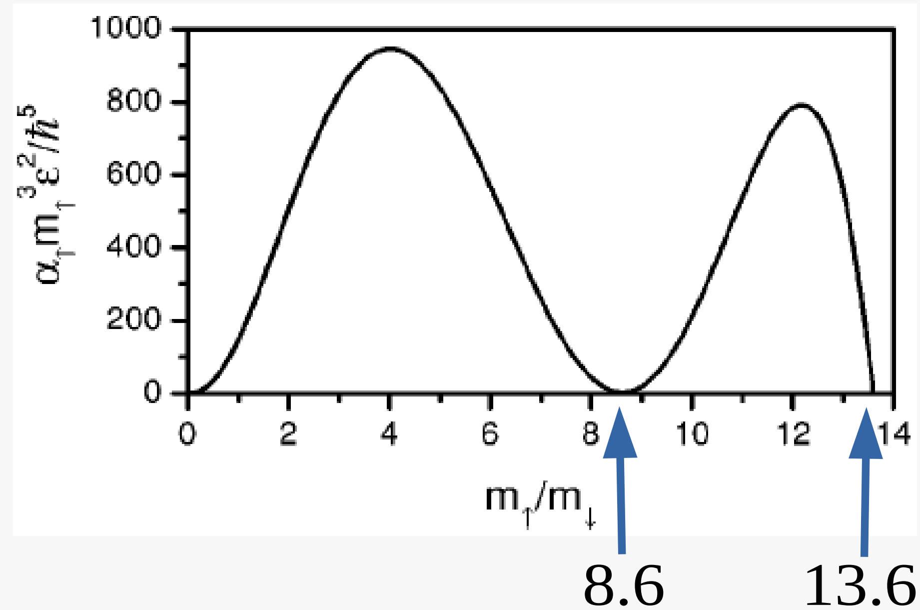
Mass-imbalanced fermionic mixtures: 4+1-body Efimov effect and universal pentamer

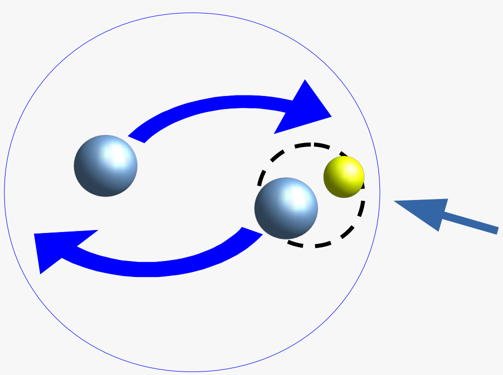
Heavy-heavy-light problem, magic mass ratios

3-body recombination to a weakly bound level DSP'03



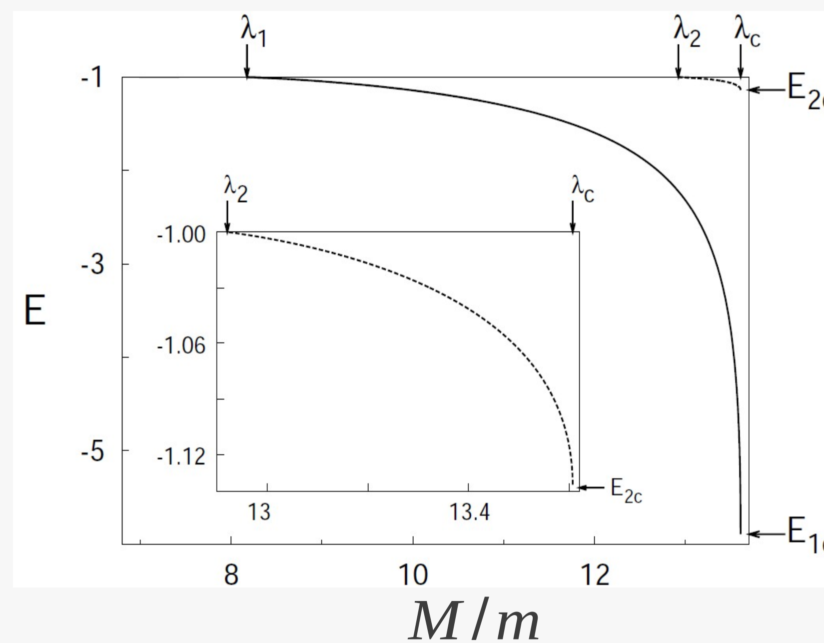


Emergence of a non-Efimovian trimer state for M/m>8.2 Kartavtsev&Malykh'06

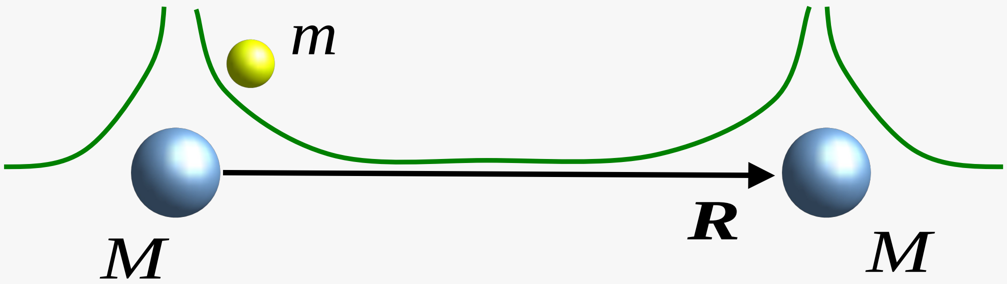


M/m<8.2 *p*-wave atom-dimer scattering resonance

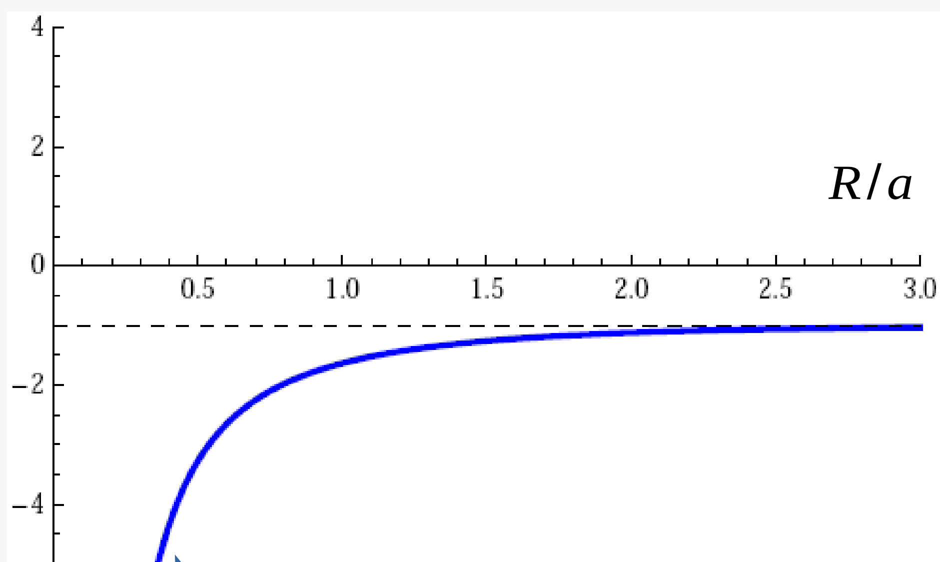
M/m>8.2 trimer state with *l*=1



Born-Oppenheimer approximation



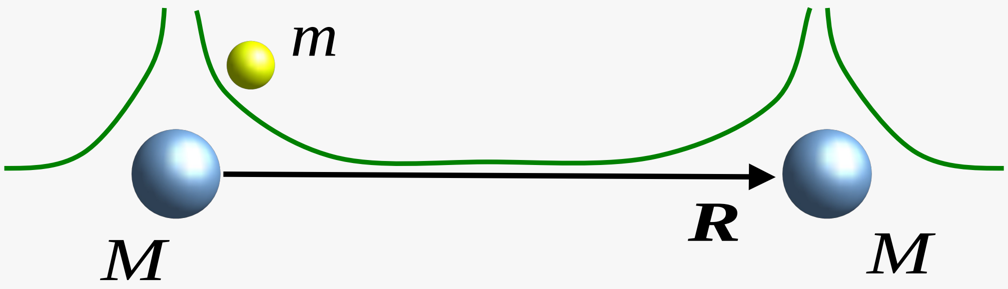
$$\frac{U_{eff}(R)}{\epsilon_0}$$



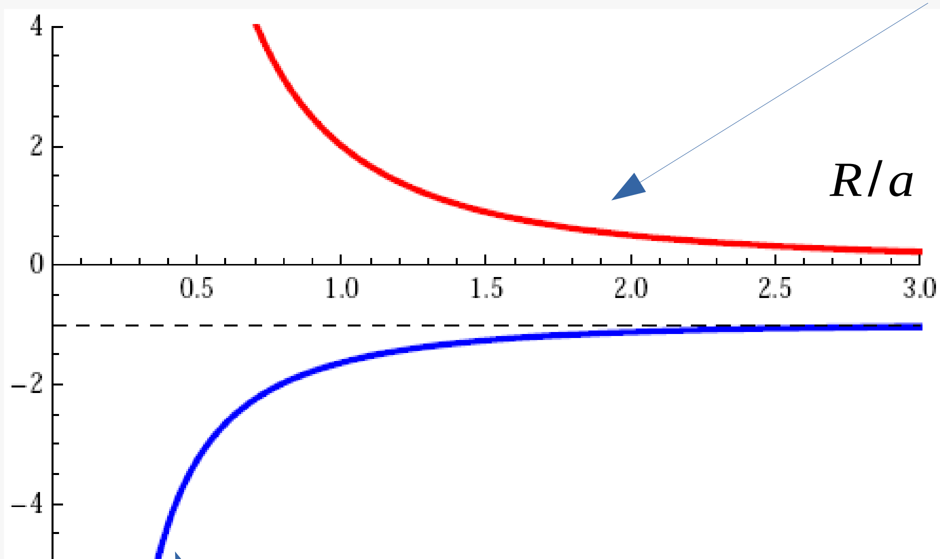
$$U_{eff}(R) \approx -\frac{\hbar^2}{2\,m\,a^2} = -|\epsilon_0|$$

$$U_{eff}(R) \approx -0.16\,\frac{\hbar^2}{m\,R^2}$$

Born-Oppenheimer approximation



$$\frac{U_{eff}(R)}{\epsilon_0}$$



$$U_{centrifugal}(R) = \frac{\hbar^2 l(l+1)}{MR^2}$$

$$U_{eff}(R) \approx -\frac{\hbar^2}{2ma^2} = -|\epsilon_0|$$

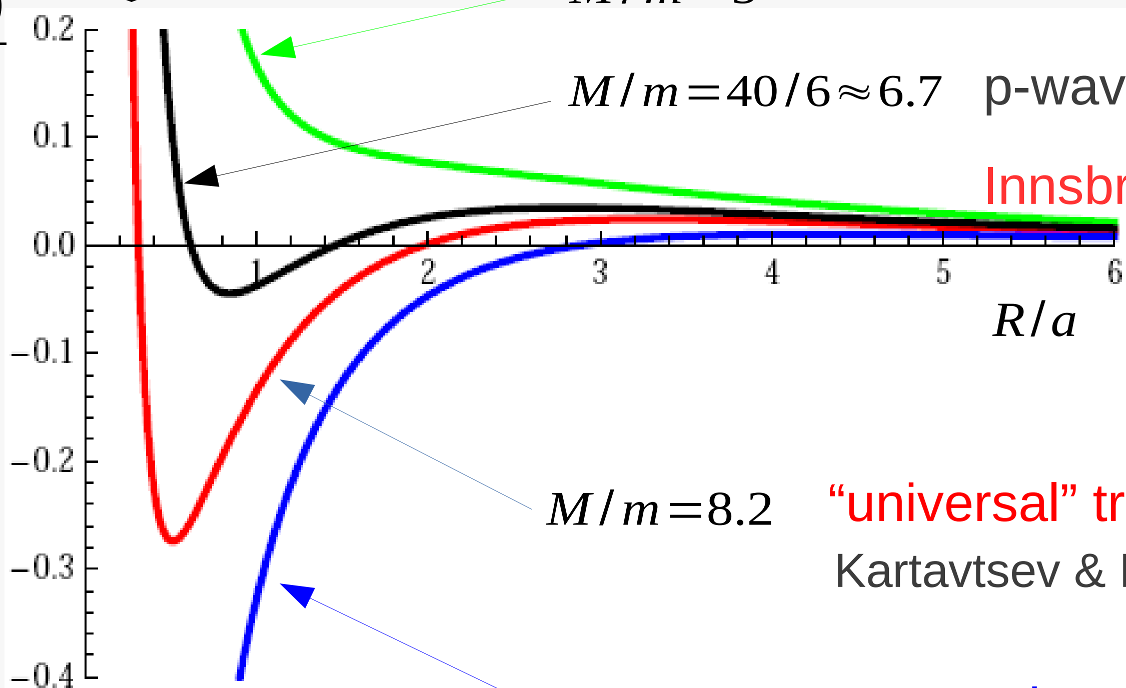
$$U_{eff}(R) \approx -0.16\frac{\hbar^2}{mR^2}$$

$$\left[-\frac{\hbar^2}{M}\frac{\partial^2}{\partial R^2}+\tilde{U}_{eff}(R)\right]\chi(R)=E\chi(R)$$

$$\frac{\hbar^2}{MR^2}\left(l(l+1)-0.16\frac{M}{m}\right)$$

$$\tilde{U}_{eff}(R)=U_{eff}(R)+|\epsilon_0|+\frac{\hbar^2 l(l+1)}{MR^2}$$

$l=1$



$M/m=5$

$M/m=40/6\approx 6.7$ p-wave scattering resonance !

Innsbruck exp: Jag et al'14

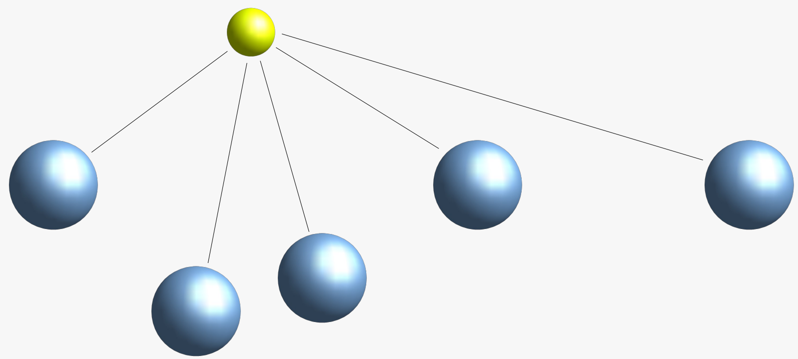
$M/m=8.2$ “universal” trimer state at threshold

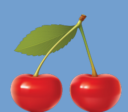
Kartavtsev & Malykh (2006)

$M/m>13.6$ many trimer (Efimov) states

(N+1)-body problem

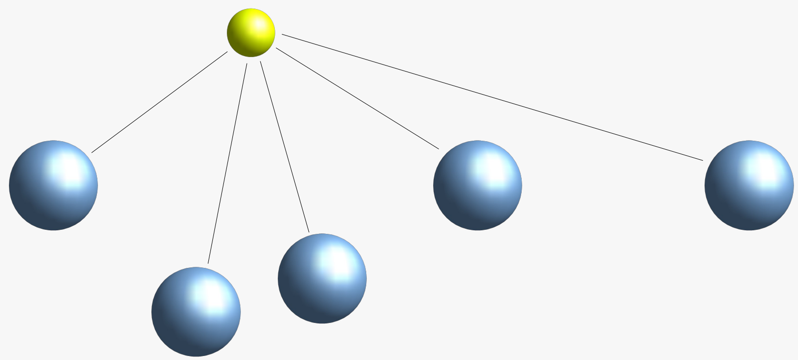
How many heavy fermions can be bound by a single light atom?

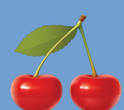
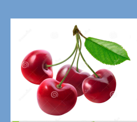


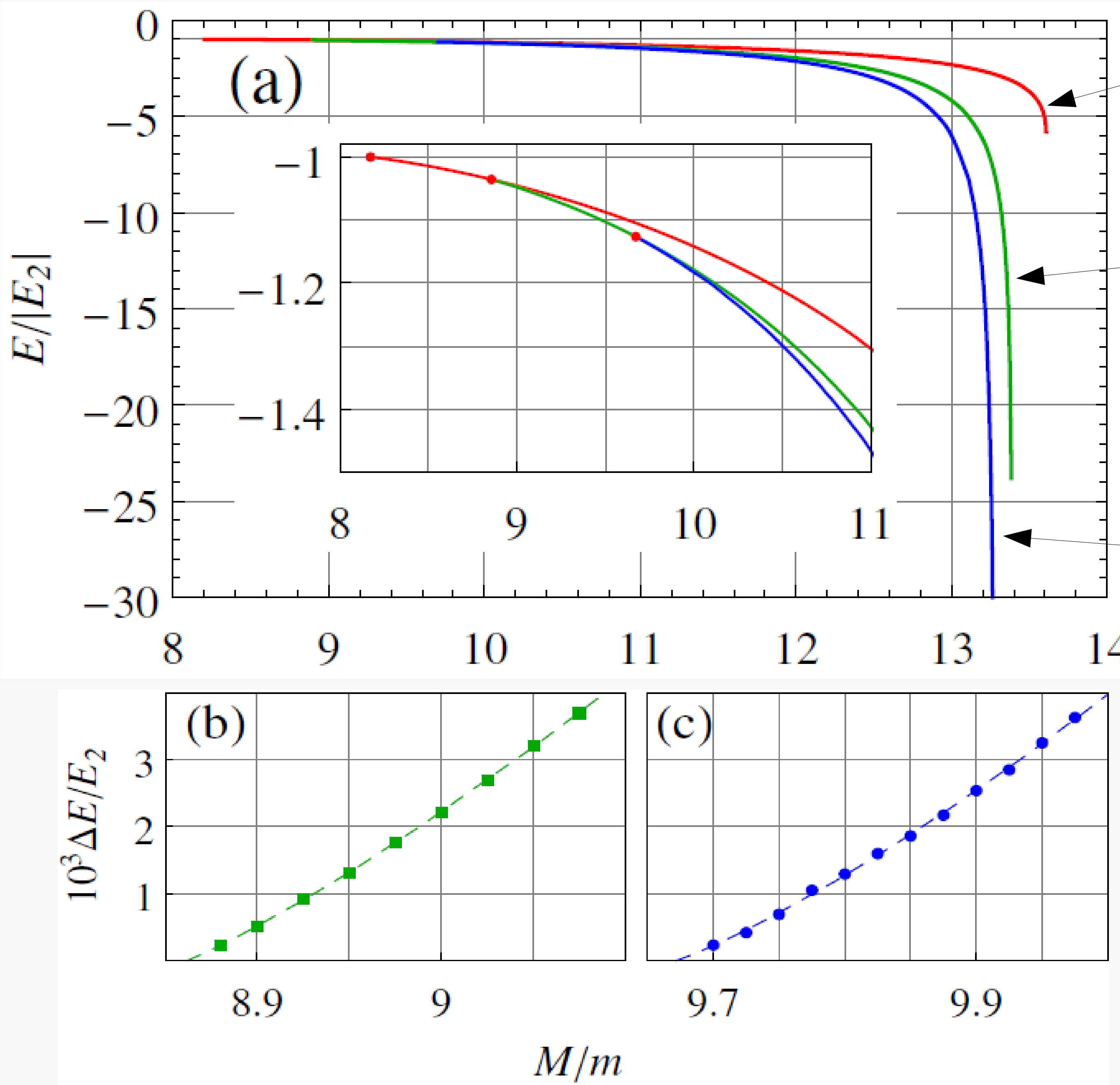
		Symmetry L^{π}	appear at $M/m>$	Efimovian for $M/m>$
2+1 trimer		1^{-}	8.173 Kartavtsev&Malykh'06	13.607 Efimov'73
3+1 tetramer		1^{+}	~9.5 Blume'12	13.384 Castin,Mora&Pricoupenko'10
4+1 pentamer		$?^{?}$	?	?
:		?	?	?
N+1-mer		?	?	?

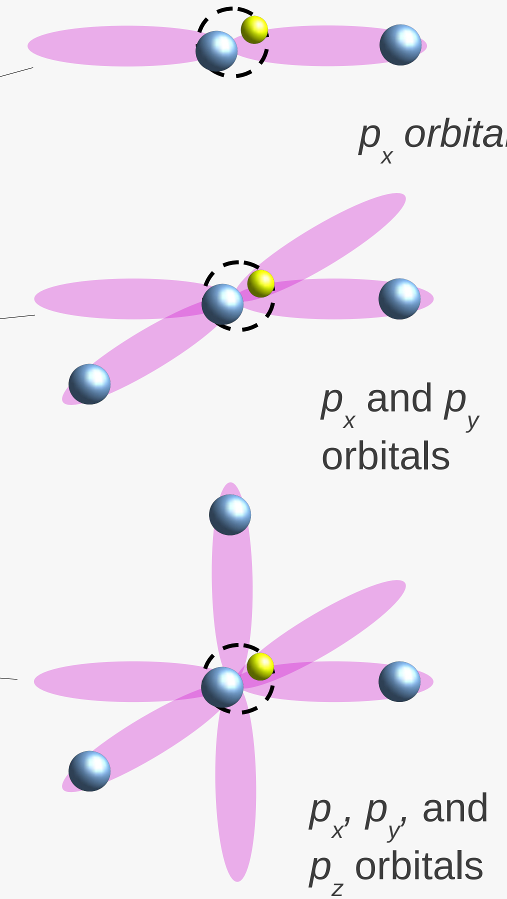
(N+1)-body problem

How many heavy fermions can be bound by a single light atom?



		Symmetry L^π	appear at $M/m>$	Efimovian for $M/m>$
2+1 trimer		1^-	8.173 Kartavtsev&Malykh'06	13.607 Efimov'73
3+1 tetramer		1^+	~9.5 → 8.862(1) Blume'12	13.384 Castin,Mora&Pricoupenko'10
4+1 pentamer		0^-	9.672(6)	13.279(2)
:		?	?	?
N+1-mer		?	?	?





pentamer = closed *p*-shell

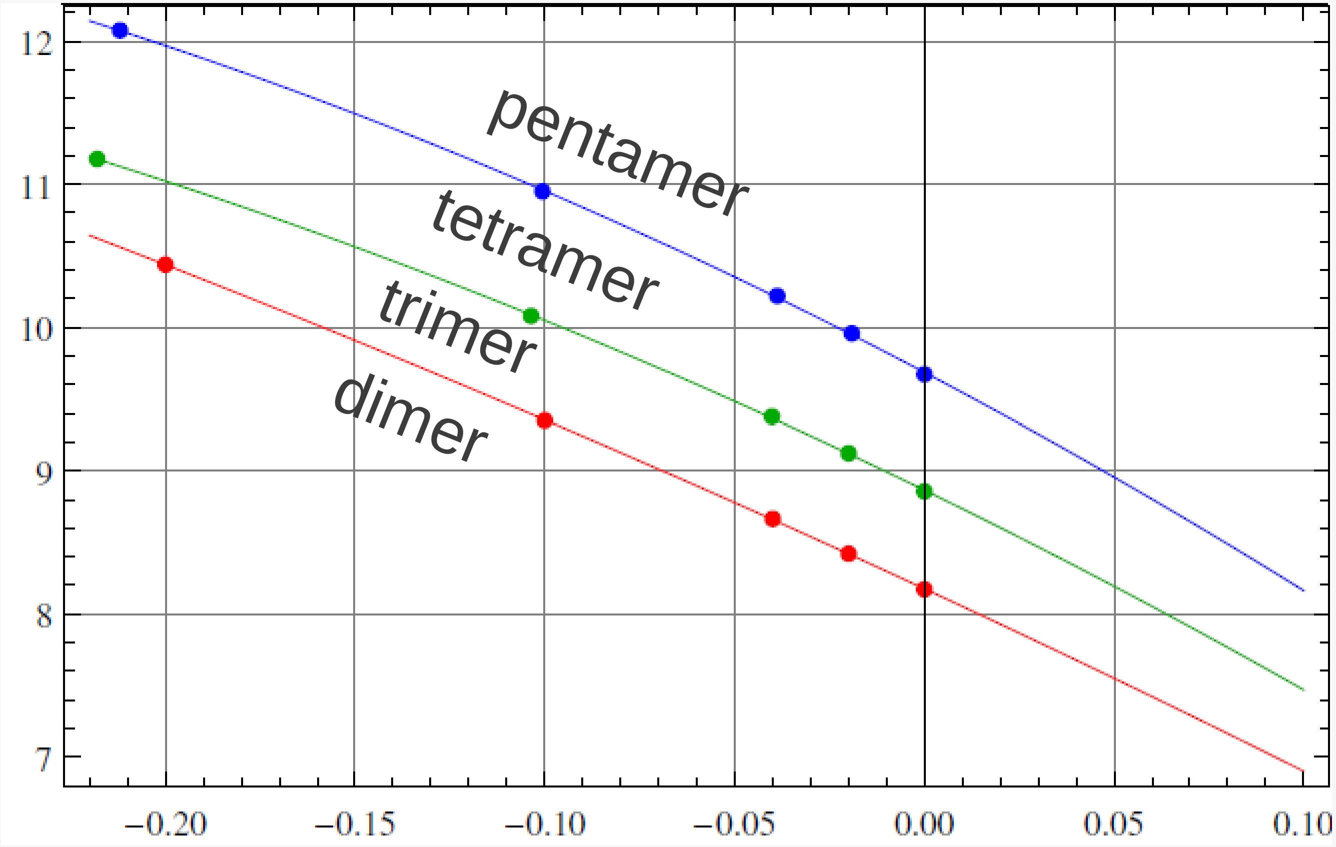
CONJECTURE:

No hexamer!

(requires justification)

Effective-range effects

$$\frac{1}{a} \longrightarrow \frac{1}{a} - \frac{r_0}{2} k^2$$

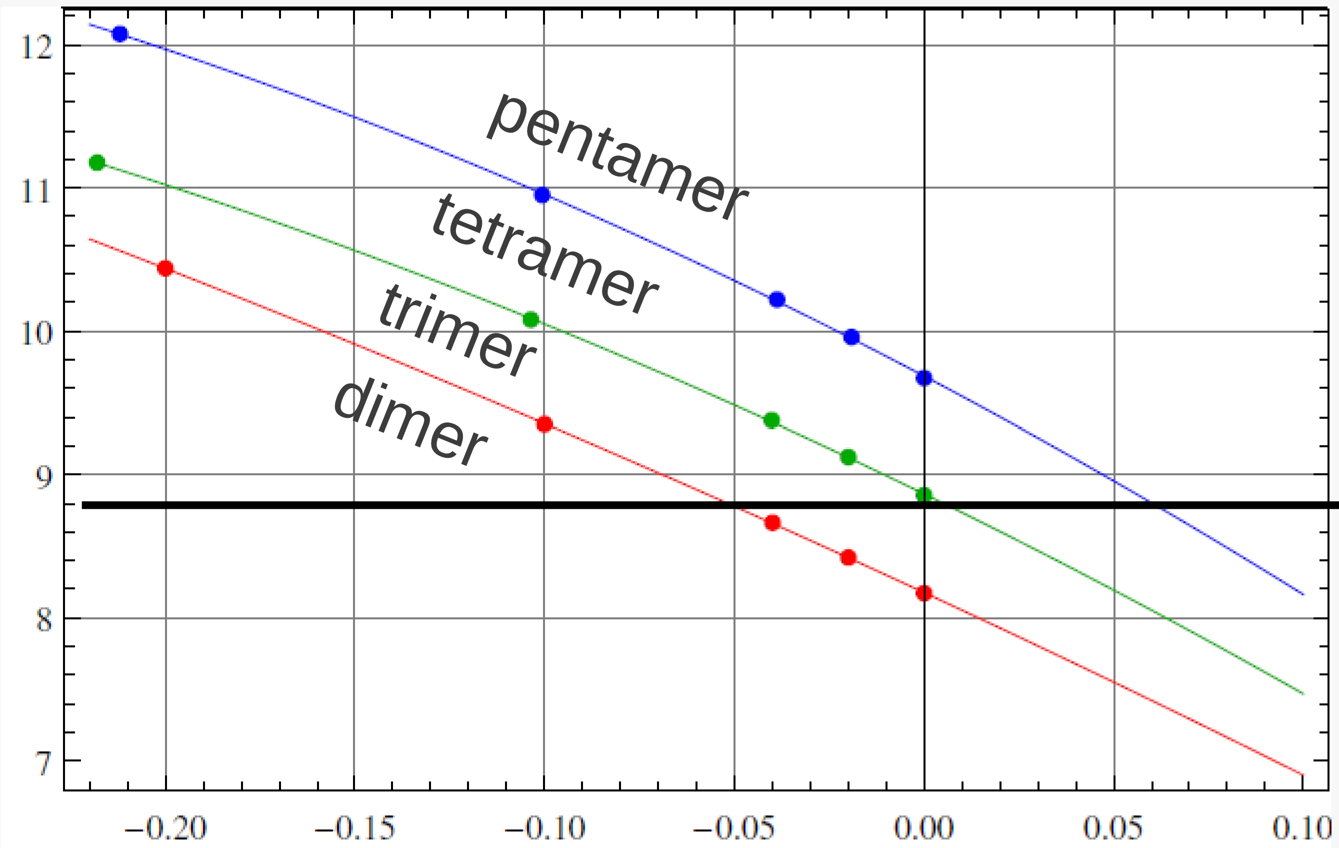


$\frac{M}{m}$

r_0 / a

Effective-range effects

$$\frac{1}{a} \longrightarrow \frac{1}{a} - \frac{r_0}{2} k^2$$



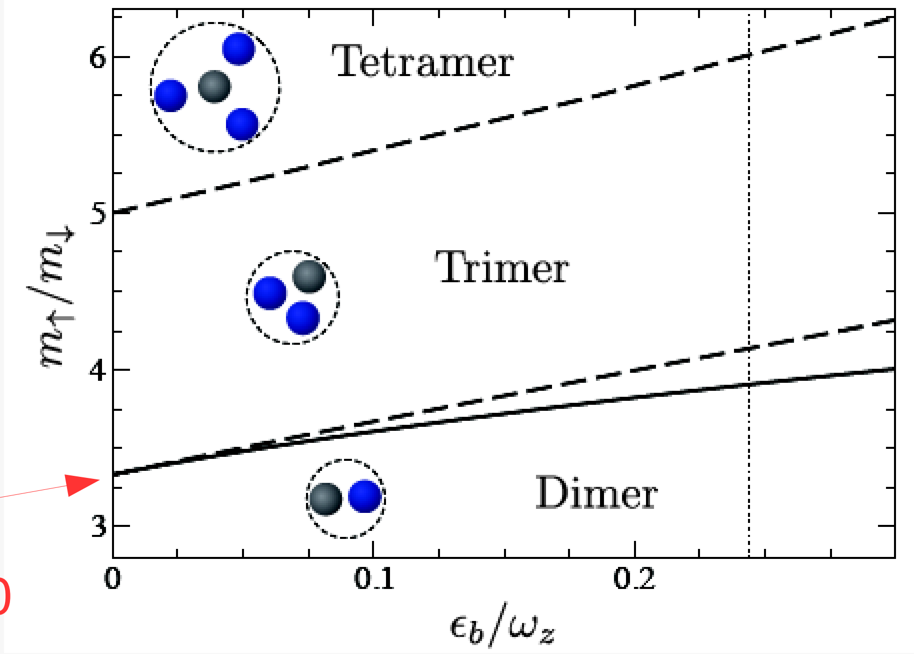
$\frac{M}{m}$

r_0/a

^{53}Cr-^{6}Li

Quasi-2D case

Levinsen&Parish'13



Pricoupenko&Pedri'10

In 2D:

- smaller mass ratio is needed

- tetramer = closed *p*-shell

Physics at *a*=infinity (& zero range)

Small-hyperradius behavior of the (*N*+1)-body wave function:

$$\left[-\frac{\partial^2}{\partial R^2} - \frac{3N-1}{R^2}\frac{\partial}{\partial R} + \frac{s^2-(3N/2-1)^2}{R^2}\right]\Psi(R) = 0$$

$$\Psi(R) \propto R^{-3N/2+1\pm s}$$

$$s^2 > 0 \quad (s>0)$$

$$\Psi(R) \propto R^{-3N/2+1+s}$$

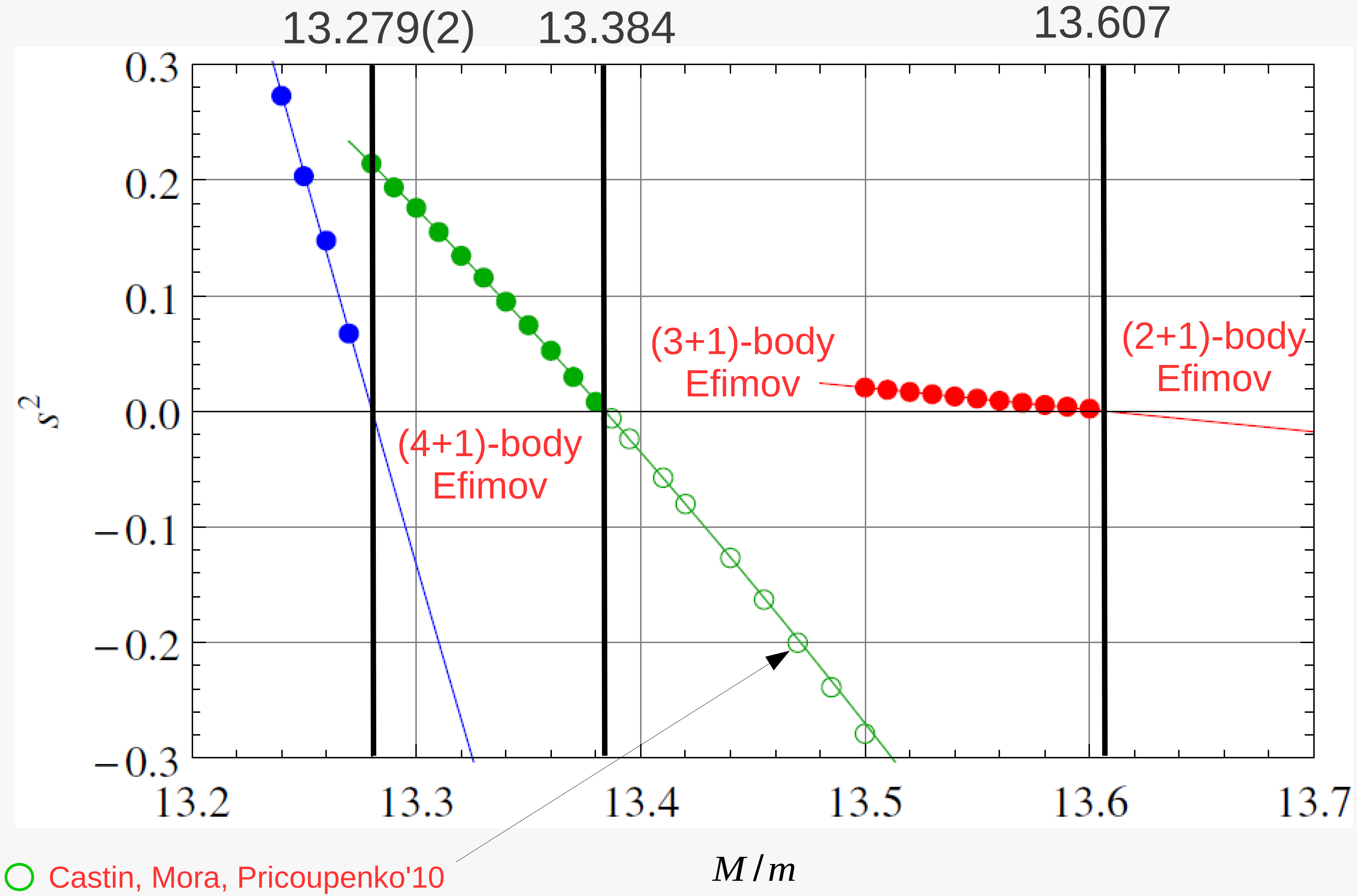
"Universal" regime in the sense that one needs no three-body parameter

Non-Efimovian regime

$$s^2 < 0 \quad (s = is_0)$$

$$\Psi(R) \propto R^{-3N/2+1} \sin(s_0 \ln R/R_0)$$

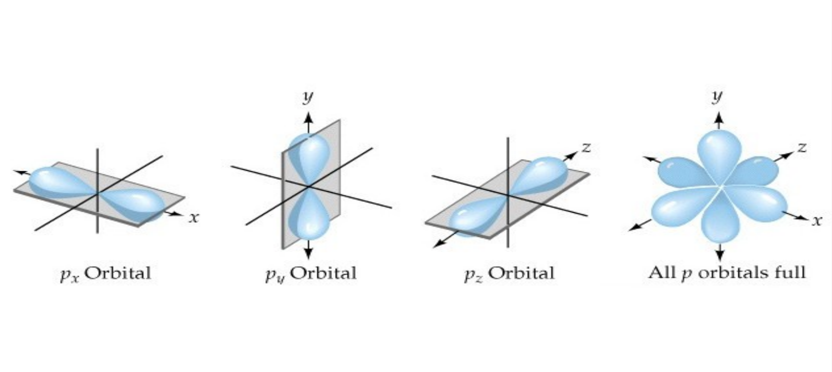
"Fall of a particle to the center in R^{-2} potential". Infinite number of zeros of the wave function. Infinite number of trimer states. Efimov effect



13.279(2)
13.384
13.607
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
s^2
(3+1)-body
Efimov
(2+1)-body
Efimov
(4+1)-body
Efimov
13.2
13.3
13.4
13.5
13.6
13.7
M / m
Castin, Mora, Pricoupenko'10

Summary

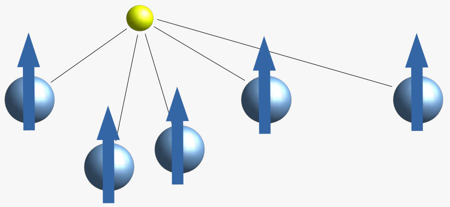
- One light atom seems to provide an almost equal binding strength to all three additional heavy fermions?!



- pentamer = closed p-shell
  - no-go theorem for hexamer and six-body Efimov effect?

- Cr-Li (M/m=8.80) promising mixture
  - many-body physics with (*N*+1)-mers

    Endo,Garcia-Garcia&Naidon'16
  - few-body: include Cr-Cr dipole interaction?



Two-dimensional bosons with zero-range interactions

2D bosons

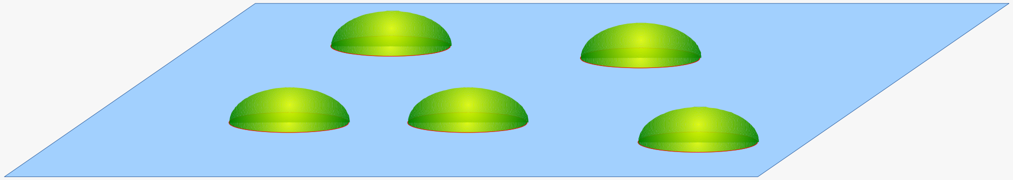
zero-range interaction = attraction

⇓

B_2 dimer energy = energy unit

$$B_3 = 16.5226874\, B_2$$

Bruch&Tjon'79; Hammer&Son'04;
Kartavtsev&Malykh'06...



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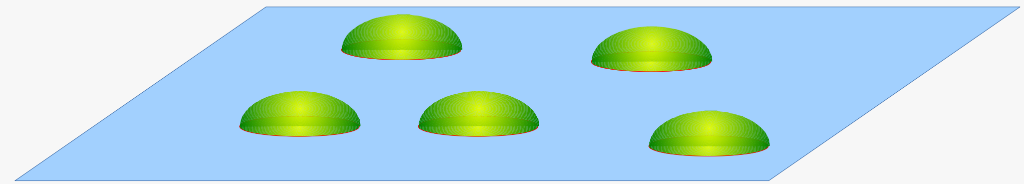


$$B_3 = 16.5226874\, B_2$$

Bruch&Tjon'79; Hammer&Son'04; Kartavtsev&Malykh'06...

Hammer&Son'04 theory in the large-N limit: $E = \frac{1}{2}\int d^2\rho \left(|\nabla \Psi|^2 + g|\Psi|^4 \right)$

$$\Psi \sim \frac{\sqrt{N}}{R} f(\rho/R)$$

N/R^2 $\quad$ $g N^2/R^2$

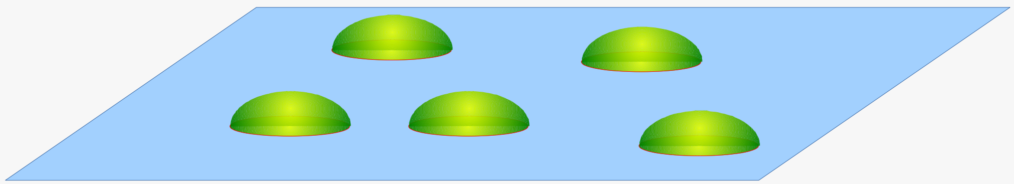
Assumption $g = 2\pi/\ln(\Lambda R)$
+ minimization wrt shape of $f(\rho)$

$$R_N \propto 0.3417^N$$
$$B_N \propto 8.567^N$$

2D bosons

zero-range interaction = attraction

B_2 dimer energy = energy unit



$B_3 = 16.5226874\, B_2$

Bruch&Tjon'79; Hammer&Son'04; Kartavtsev&Malykh'06...

Hammer&Son'04 theory in the large-N limit: $E = \frac{1}{2}\int d^2\rho \left(|\nabla \Psi|^2 + g|\Psi|^4\right)$

$\Psi \sim \frac{\sqrt{N}}{R} f(\rho/R)$

N/R^2 $\qquad$ $g N^2/R^2$

$B_4 = 197.3(1)\, B_2$

Platter,Hammer&Meissner'04; Brodsky et al'06

$N \le 7$ finite-range calculations
Blume'05 (inconclusive)

$N \le 10$ lattice EFT
Lee'06 $\quad B_N / B_{N-1} \rightarrow 8.3(6)$

Assumption $g = 2\pi/\ln(\Lambda R)$
+ minimization wrt shape of $f(\rho)$

$R_N \propto 0.3417^N$

$B_N \propto 8.567^N$

Few-to-many body crossover question remains open !

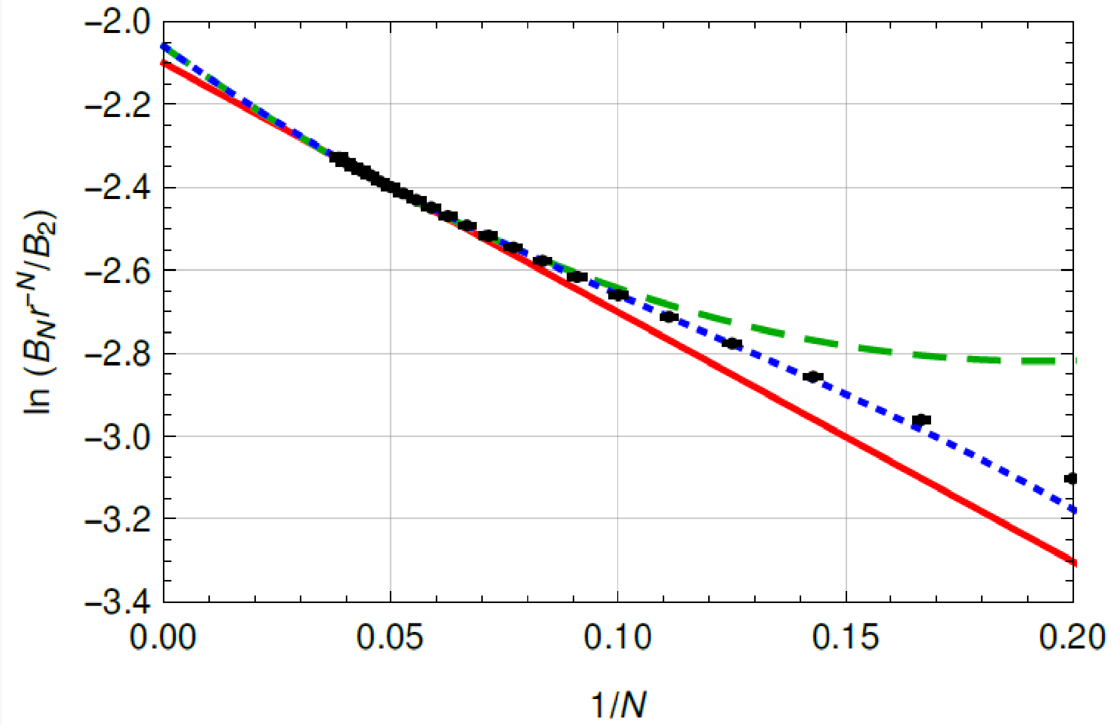
$$B_N = B_2\, e^{\ln(8.567) N + c_1 + c_2/N + \ldots}$$

Our results

N	B_N/B_2	N	B_N/B_2
3	$1.65225(2)\times10^{1}$	15	$8.135(2)\times10^{12}$
4	$1.9720(1)\times10^{2}$	16	$7.129(4)\times10^{13}$
5	$2.0745(1)\times10^{3}$	17	$6.232(2)\times10^{14}$
6	$2.0471(1)\times10^{4}$	18	$5.438(3)\times10^{15}$
7	$1.9462(1)\times10^{5}$	19	$4.734(2)\times10^{16}$
8	$1.8070(1)\times10^{6}$	20	$4.119(2)\times10^{17}$
9	$1.6508(4)\times10^{7}$	21	$3.577(2)\times10^{18}$
10	$1.4905(2)\times10^{8}$	22	$3.108(4)\times10^{19}$
11	$1.3345(2)\times10^{9}$	23	$2.694(5)\times10^{20}$
12	$1.1873(4)\times10^{10}$	24	$2.332(4)\times10^{21}$
13	$1.0508(3)\times10^{11}$	25	$2.018(4)\times10^{22}$
14	$9.2596(9)\times10^{11}$	26	$1.748(4)\times10^{23}$

$$B_N = B_2 e^{\ln(8.567)N + c_1 + c_2/N + \dots}$$

$$\ln\left(B_N 8.567^{-N}/B_2\right) = c_1 + c_2/N + \dots$$



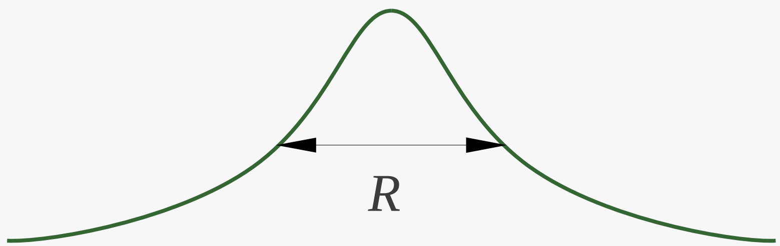
Fits: $\{c_1, c_2, \dots\} = \{-2.1, -6.01\}$

$\{-2.06, -7.88, 20.45\}$

$\{-2.06, -7.94, 27.2, -77\}$

Prospects

- beyond-Hammer&Son theory = Bogoliubov theory in the inhomogeneous case, i.e., beyond LDA, since healing length ~ droplet size



$$n \sim N/R^2$$

$$g \sim 1/\ln(\Lambda R_N) \sim 1/N \ll 1$$

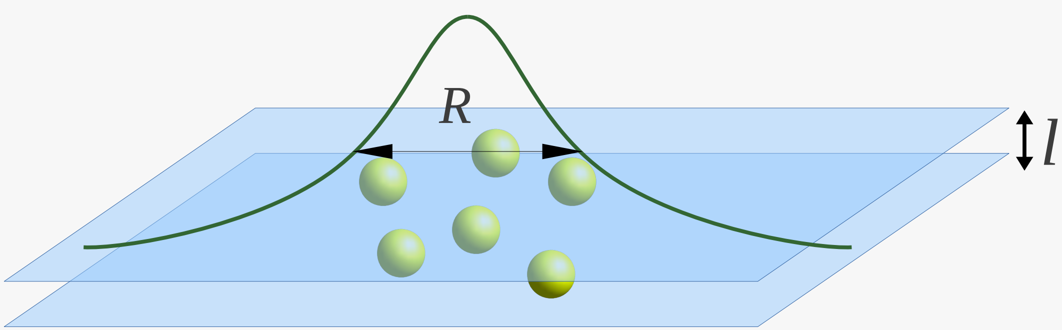
$$\xi = 1/\sqrt{gn} \sim R$$

- theory for dynamics + excitations for large *N*?

  excited trimer and tetramer states are known Bruch&Tjon'79; Platter et al'04; Brodsky et al'06

  our method (so far) does not work for excited states :(

- experimental realization: droplets with *N*~10 to 100 are realistic in quasi 2D



$$l \ll R \sim l\, e^{\sqrt{\pi/2}\, l/|a| - N \ln\sqrt{8.576}} < \text{trap size}$$

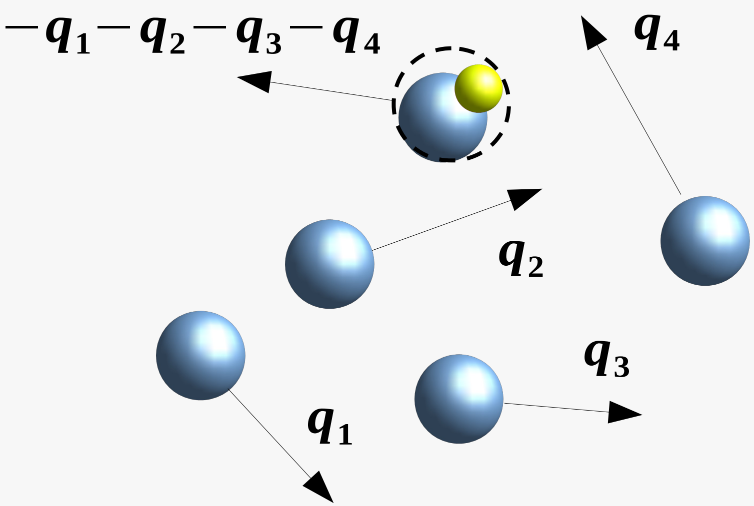
Method of calculations (STM-DMC)

STM part

(*N*+1)-body Skorniakov – Ter-Martirosian equation (STM) [Pricoupenko'11]:

$$\frac{1}{4\pi}\left(\frac{1}{a}+\frac{r_0\kappa^2}{2}-\kappa\right)F(\mathbf{q}_1,\ldots,\mathbf{q}_{N-1})=\int\frac{d^3q_N}{(2\pi)^3}\frac{\sum_{i=1}^{N-1}F(\mathbf{q}_1,\ldots,\mathbf{q}_{i-1},\mathbf{q}_N,\mathbf{q}_{i+1},\ldots,\mathbf{q}_{N-1})}{-\frac{2\mu E}{\hbar^2}+\frac{\mu}{M}\sum_{i=1}^{N}q_i^2+\frac{\mu}{m}\left(\sum_{i=1}^{N}\mathbf{q}_i\right)^2}$$

where $\kappa=\sqrt{-\frac{2\mu E}{\hbar^2}+\frac{\mu}{M}\sum_{i=1}^{N-1}q_i^2+\frac{\mu}{M+m}\left(\sum_{i=1}^{N-1}\mathbf{q}_i\right)^2}$



N=2: $F(\boldsymbol{q}_1)=\hat{\boldsymbol{q}}_1\cdot\hat{z}\,f(q_1)$

N=3 [Castin, Mora, Pricoupenko'10]:

$$F(\boldsymbol{q}_1,\boldsymbol{q}_2)=\hat{z}\cdot\hat{\boldsymbol{q}}_1\times\hat{\boldsymbol{q}}_2\,f(q_1,q_2,\boldsymbol{q}_1\cdot\boldsymbol{q}_2)$$

N=4:

$$F(\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3)=\hat{\boldsymbol{q}}_1\cdot\hat{\boldsymbol{q}}_2\times\hat{\boldsymbol{q}}_3\,f(q_1,q_2,q_3,\boldsymbol{q}_1\cdot\boldsymbol{q}_2,\boldsymbol{q}_1\cdot\boldsymbol{q}_3,\boldsymbol{q}_2\cdot\boldsymbol{q}_3)$$

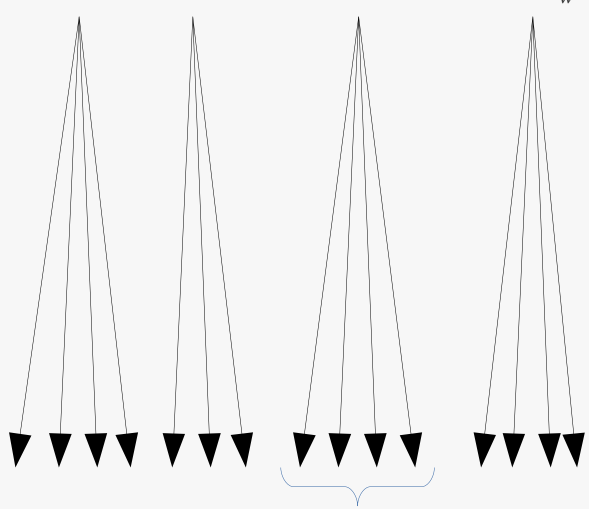
Advantages of STM (versus Schroedinger):
- zero-range interactions are treated naturally
- removes three coordinates
- reduces the problem to symmetric *f* (at least, for *N*<5)

DMC part

f – symmetric → ground state → *f*>0 → dens. distr. function → organize a diffusion process for which STM is the detailed balance equation

STM equation: $$f(\vec{Q}) = \int K(\vec{Q}, \vec{Q}')\, f(\vec{Q}')\, d^{3(N-1)} Q'$$

Iteration # j $$\{\vec{Q}_1, \vec{Q}_2, \ldots, \vec{Q}_i, \ldots, \vec{Q}_{N_w}\}$$



each walker is assigned weight

$$W(\vec{Q}_i) = \int K(\vec{Q}, \vec{Q}_i)\, d^{3(N-1)} Q$$

according to which it is branched and the new position of each child is drawn from

$$PDD(\vec{Q}_i) = K(\vec{Q}, \vec{Q}_i) / W(\vec{Q}_i)$$

Iteration # j+1 $$\sim \sum_i W(\vec{Q}_i)$$ new walkers

Detailed balance equation for this process is the STM equation!

Requirements:
- fast branching/sampling scheme (OK, the structure of STM is simple)
- $f(Q)$ should be normalizable (if not, introduce a weight function)

Overall characteristics

+ works directly in the zero-range limit (extrapolation procedure not needed)

+ can treat large configurational spaces

+ can be generalized to mixtures, to mixed-dimensional systems, to unitary trapped case, etc.

- extrapolation in the number of walkers can become necessary for larger N

    solution: use large number of walkers :) possible because of relatively small thermalization time of the algorithm

- zero range cannot model repulsive interactions (for D>1)

    possible solution: remove the high-momentum pole of the corresponding scattering amplitude, extrapolate from weak attraction to weak repulsion, etc.

- SIGN PROBLEM! the method cannot automatically determine nodes of the wave function (they should be known in advance or assumed). Examples: 5+1-body fermions (hexamer), Efimov states, excited states, etc.

    possible solution (usual stuff): fixed nodes, annihilation of walkers, semi-deterministic methods based on finite grid, etc.

± requires ad hoc approaches to accelerate sampling and branching for each particular system, the equilibrium walker distribution should be normalizable (requires a weight function and some knowledge of underlying physics)

THANK YOU!